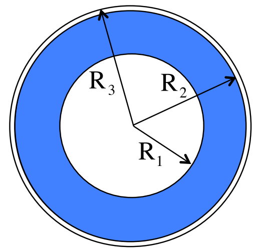
Problem 10.31

A tire can be modeled as three pieces, two side-walls and one tread pad.

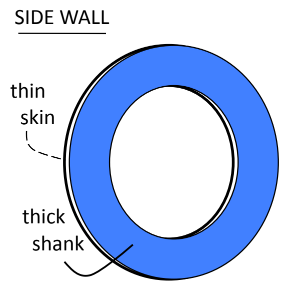
The side-wall: If you took a pair of heavy duty scissors and cut one of the side-walls away from the rest of the tire, you would end up with a very thin skinned, thick shanked cylinder (see sketch).



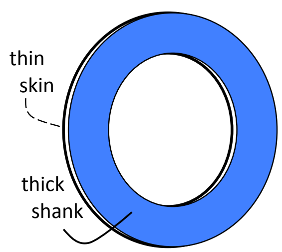
Although I always like to derive my expressions (something *you* should be able to do for a cylinder), the text's Solution Manual seems to think that you can start with the *moment of inertia* of a cylinder as a given quantity. That expression is:

$$I_{cyl} = \frac{1}{2} m_{cyl} \left(R_1^2 + R_2^2\right)$$

where R_1 is the inside radius and R_2 is the outside radii *not of the tire* but, rather, the outside radius of the cut swath.



We are given the *volume mass density* of the rubber (ρ), so knowing the volume of the side-wall, we can determine its mass. Noting that volume is *surface area* times *thickness* "t", where the surface area is



$$\pi R_2^2 - \pi R_1^2$$

we can write the mass as:

$$m_{sideWall} = \rho\left(\left(\pi R_2^2 - \pi R_1^2\right) t_{wall}\right)$$

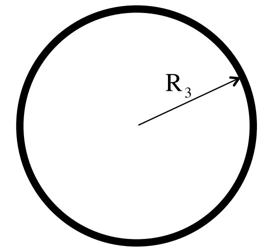
With two side-walls, the net *moment of inertia* due to both will be (note that the last line is a simplification of the R terms done to save space):

$$I_{2sideWalls} = 2\left[\frac{1}{2} \qquad m_{sideWall} \qquad \left(R_1^2 + R_2^2\right)\right]$$

$$= 2\left[\frac{1}{2}\left[\rho\left(\left(\pi R_2^2 - \pi R_1^2\right) t_{wall}\right)\right]\left(R_1^2 + R_2^2\right)\right]$$

$$= 2\left[\frac{1}{2}\left[\rho\left(\pi\left(R_2^4 - R_1^4\right) t_{wall}\right)\right]\right]$$

As for the relatively THIN tread pad, because it is so thin it is tempting to assume that all of the mass is located a distance R_3 meters from the center and go from there. In that scenario, and remember what the *moment of inertia* asks us to do, we would go out from the axis in question until we found our mass at the assumed position R_3, then multiply that mass by the square of that distance. This would lead to a *moment of inertia* of the tread pad of:



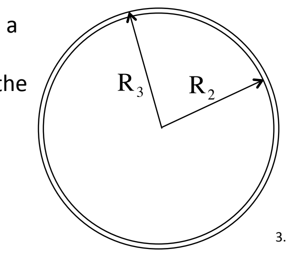
$$I_{treadPad} = m_{pad} R_3^2$$

(Why? Because all of the mass would be assumed to be a distance R_3 out.)

The problem with this is that in reality, the pad is also a cylinder with an inside and outside radius. In other words, the *moment of inertia* for it should also be of the form:

$$I_{pad} = \left[\frac{1}{2}\left[\rho\left(\pi\left(R_3^4 - R_2^4\right) t_{pad}\right)\right]\right]$$

where the "r" values are defined on the sketch.



Using that while omitting units and sig figs to conserve space, the total *moment of inertia* for the entire structure then becomes:

$$I = \cancel{2}\left[\frac{1}{\cancel{2}}\left[\rho\left(\pi\left(R_2^4 - R_1^4\right) t_{wall}\right)\right]\right] + \left[\frac{1}{2}\left[\rho\left(\pi\left(R_3^4 - R_2^4\right) t_{pad}\right)\right]\right]$$

$$= \left[\left[\left(1.1\text{x}10^3\right)\pi\left((.305)^4 - (.165)^4\right)\left(6.35\text{x}10^{-3}\right)\right]\right]$$

$$+ \left[\frac{1}{2}\left[\left(1.1\text{x}10^3\right)\pi\left((.33)^4 - (.305)^4\right)(.2)\right]\right]$$

$$= \left[\left(1.1\text{x}10^3\right)\pi\right]\left(\left[\left((.305)^4 - (.165)^4\right)\left(6.35\text{x}10^{-3}\right)\right] + \left[\frac{1}{2}\left[\left((.33)^4 - (.305)^4\right)(.2)\right]\right]\right)$$

$$= 1.28 \text{ kg} \bullet \text{m}^2$$

The moral of the story? When trying to determine the *moment of inertia* of a system that has many pieces to it, determine the *moment of inertia* due to EACH piece, then add them all together.